26. $f(t) = \frac{2}{1-t^2}$ $\quad g(x) = \int_0^x f(t)\,dt$

a) $\frac{a}{1-r}$ geometric $a = 2$ $r = t^2$

$$f(t) = \int_0^x \frac{2}{1-t^2} = \int_0^x 2 + 2t^2 + 2t^4 + 2t^6 \ldots 2t^{2n}$$

$-1 < t < 1$

b) $g(t)\Big|_0^x = 2t + \frac{2t^3}{3} + \frac{2t^5}{5} + \frac{2t^7}{7} + \cdots \Big|_0^x$

$$g(x) = 2x + \frac{2x^3}{3} + \frac{2x^5}{5} + \frac{2x^7}{7} \ldots$$

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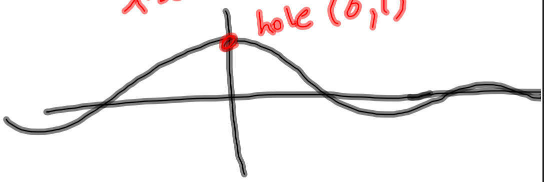
43. $f(x) = \frac{\sin x}{x}$

$\lim\limits_{x \to 0} \frac{\sin x}{x} = \lim \frac{\cos x}{1} = 1$

hole $(0,1)$



a) $\frac{\sin x}{x} = \frac{x}{x} - \frac{x^3}{x \cdot 3!} + \frac{x^5}{x \cdot 5!} - \frac{x^7}{x \cdot 7!} \ldots$

$$\frac{\sin x}{x} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} \ldots (-1)^n \frac{x^{2n}}{(2n+1)!}$$

if $x \neq 0$

c) $\begin{cases} \frac{\sin x}{x} & \text{if } x \neq 0 \\ 1 & \text{if} = 0 \end{cases}$

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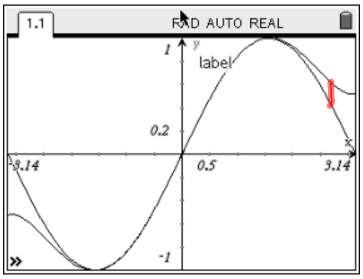
9.3 Taylor's Remainder Estimation Theorem

$$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} \quad \text{on } [-\pi, \pi]$$

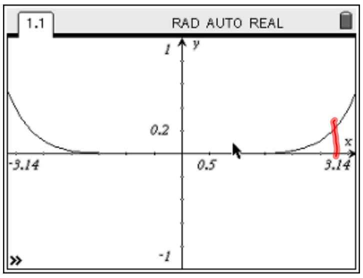
$$\text{error} = \left| \underbrace{\sin x}_{f1(x)} - \underbrace{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!}\right)}_{f2(x)} \right|$$



$$\text{error} = |f1(x) - f2(x)|$$

error function $f3(x)$

error is the distance between the two



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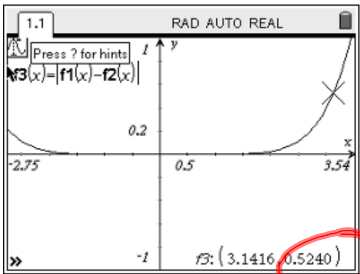
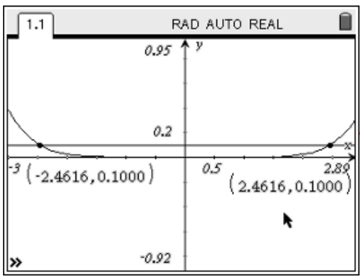
What is the max error on $[-\pi, \pi]$



= max error

on what interval is the max error = .1



$[-2.4616, 2.4616]$

$\leftarrow f4(x) = .1$

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Taylor Error estimation Theorem

If $f(x) \approx f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots + \frac{f^n(a)}{n!}(x-a)^n$

$$\boxed{\text{error} \le \left| \frac{M(x-a)^{n+1}}{(n+1)!} \right|}$$

M is an upper bound for $f^{n+1}(x)$

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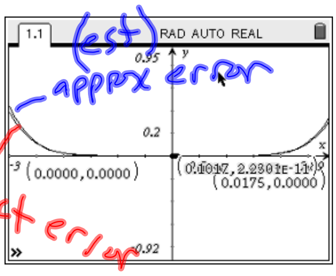
$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!}$

exact error = $\left| \sin x - \left(x - \frac{x^3}{3!} + \frac{x^5}{5!}\right) \right|$ f3(x)

est. error = $\left| \frac{M x^7}{7!} \right|$ f4(x) $\quad M = 1$

$= \left| \frac{x^7}{7!} \right| \qquad \frac{d^7 \sin x}{dx^7} \le 1$



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5 terms

$n=0,\ n=1,\ \dots\ n=4$

How many terms are needed to approx

sinx with a max error of .05

on $[-\pi, \pi]$?

$$\sin x = x - \frac{x^3}{3!} + \cdots + \frac{x^{2n+1}}{(2n+1)!}(-1)^n$$

solve for n

$$\text{error} \le \left| \frac{x^{2n+3}}{(2n+3)!} \cdot M \right| \le .05 \qquad M = 1$$

let $x = \pi$

solve for n

$$\frac{\pi^{2n+3}}{(2n+3)!} \le .05$$

$\boxed{n=4}$

$n=5$

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